Some Approximations in the Nonlinear Vibrations of Unsymmetrically Laminated Plates

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Theme

By combining a linear approximation due to Ashton for unsymmetric laminates and a nonlinear approximation due to Berger for isotropic nonlinear plates, an approximation to the equations describing the nonlinear vibrations of unsymmetric angle ply laminates is obtained. Solutions are obtained for hinged plates and compared to solutions of the exact equation.

Content

If a solution

$$w(x, y) = b \sum_{m=1}^{M} \sum_{n=1}^{N} \xi_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (1)$$

is assumed to the dynamic von Kármán equations for angle ply laminates a Galerkin procedure will lead to the following set of equations for the first four modes[1]

$$\ddot{\xi}_i + F_{0i}\xi_i + D_i\xi_a\xi_b\xi_c + J_i\xi_d\xi_e + \sum_{n=1}^{4} G_{iin}\xi_i\xi_n^2 = F_i(t) \quad (2)$$

$$a \neq b \neq c \neq i \qquad i + d + e = 6 \qquad d,e \neq i$$

The bending stretching coupling terms enter only the linear frequency terms and the quadratic nonlinear term. The quadratic terms are in general small. The derivation of these equations is algebraically complex and tedious even for a single degree of freedom and simple mode shapes. An approximation due to Berger[2] and extended by Wah[3] has proved useful in simplifying the analysis of nonlinear isotropic and orthotropic plates. Unfortunately, it was shown that for angle ply laminates the bending stretching terms render this approximation infeasible.[1] An approximation suggested by Ashton[4] for linear problems involves neglecting certain of the bending stretching coupling terms in such a way that the application of the Berger approximation becomes feasible. Combining these approximations and using solution (1), the time dependent equations become

$$\ddot{\xi}_{mn} + \frac{\pi^4\gamma^2}{r^4 E_{11}}\{\bar{D}_{11}^* m^4 + 2(\bar{D}_{12}^* + 2\bar{D}_{66}^*)r^2m^2n^2 + \bar{D}_{22}^* r^4 n^4\}\xi_{mn} + \frac{\pi^4 \xi_{mn}}{8E_{11}r^4}\sum_{i=1}^{M}\sum_{j=1}^{N} \xi_{ij}^2 \frac{\bar{A}_{22}^* m^2 i^2 + \bar{A}_{11}^* r^4 n^2 j^2 + (\bar{A}_{22}^*\bar{A}_{11}^*)^{1/2} r^2 (m^2 j^2 + n^2 i^2)}{\bar{A}_{11}^*\bar{A}_{22}^* - \bar{A}_{12}^{*2}} = F_{mn}(x, \tau) \quad (3)$$

where

$$\tau = (E_{11}/\rho)^{1/2} t/h$$

$$\bar{A}_{ij}^* = A_{ij}^* h; \qquad \bar{B}_{ij}^* = B_{ij}^*/h; \qquad \bar{D}_{ij}^* = D_{ij}^*/h^3$$

$$\gamma = h/b, \quad r = a/b$$

and A_{ij}^*, B_{ij}^*, D_{ij}^*, and E_{11} are material constants.

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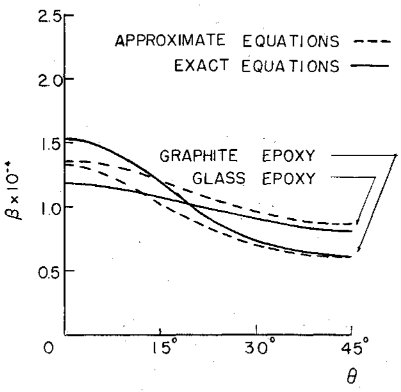

Fig. 1 Variation of nonlinearity β with ply orientation θ, orthotropic case, $\gamma = 0.01$, $r = 1.0$.

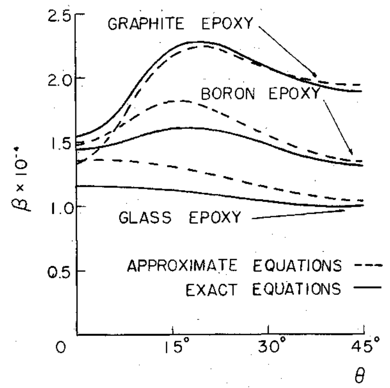

Fig. 2 Variation of nonlinearity β with ply orientation θ, 2 ply case, $\gamma = 0.01$, $r = 1.0$.

The linear frequency term does not contain the bending stretching coupling terms $\bar{B}^*$ explicitly as in the von Kármán solution and the quadratic nonlinear terms are not present.

For first mode response to harmonic forcing the equation reduces to

$$\ddot{\xi}_1 + F_{01}\xi_1 + G_{111}\xi_1{}^3 = P_0 \cos \omega t \quad (4)$$

and the amplitude frequency relation for harmonic motion is

$$(\omega/\omega_0)^2 = 1 + \beta A_1{}^2 - P_0/A_1; \qquad \beta = \tfrac{3}{4}G_{111}/F_{01}$$

Comparisons between the solutions of the approximate and von Kármán equations are shown in Figs. 1–3. In all case $\gamma = 0.01$. Previously published material constants are used.[1] For the orthotropic case the linear or Ashton approximation does not effect the results since $B_{ij} = 0$ in this case. Comparison of Figs. 1 and 2 indicate that in the 2 ply case the effect of the linear approximation is small compared to the effect of the nonlinear approximation. Figure 3 indicates that the approximation becomes quite inaccurate as a/b changes from unity. These errors are clearly due to the nonlinear approximation.

The approximation has also dropped the quadratic coupling terms. Since the quadratic terms couple the modes, their primary effect will be to excite nonresonant modes through parametric excitation. The case of $F_2 = F_3 = F_4 = 0$ and ξ_1 in a region

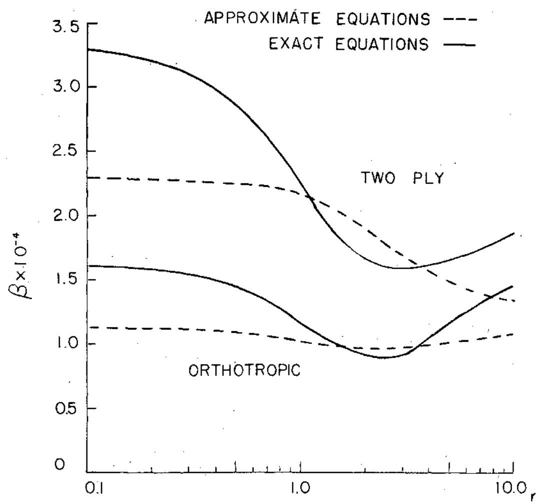

Fig. 3 Variation of nonlinearity β with aspect ratio r, graphite epoxy $\theta = 15°$, $\gamma = 0.01$.

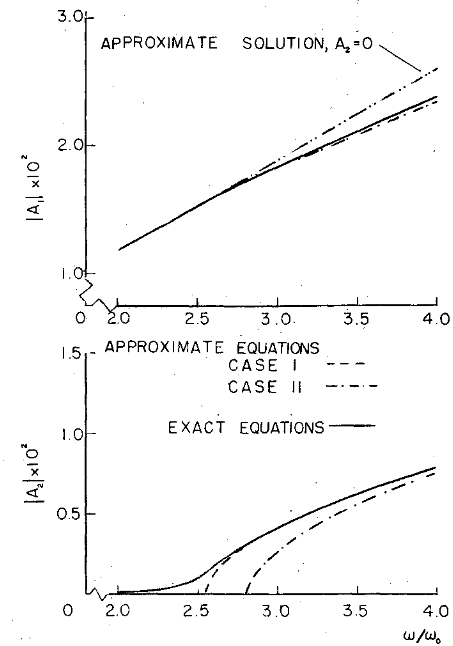

Fig. 4 Mode 1 forced, solutions for modes 1 and 4, exact coefficients, graphite epoxy, $\theta = 15°$, $\gamma = 0.01$, $P_0 = 10^{-6}$.

where ξ_4 may be excited by parametric excitation is investigated. Two possible solutions exist for the equations if the quadratic term J_4 is neglected

case I $\xi_1 = A_1 \cos \omega t$ case II $\xi_1 = A_1 \cos \omega t$

$\xi_4 = d_2 + A_2 \cos 2\omega t$ $\xi_4 = A_2 \sin 2\omega t$

If J_4 is included, harmonic balance techniques indicate that only the case I solution is possible.

In order to demonstrate the effect of neglecting J_4 the case I and case II solutions, neglecting J_4, and the case I solution including J_4 are shown in Fig. 4 for $P_0 = 10^{-6}$. For purposes of comparison the exact values of the constants are used. The spurious solution, case II must be ignored in any approximate solution. If J_4 is included, the second is excited for all values of ω/ω_0, whereas if J_4 is neglected, the second mode is excited for all values of $\omega/\omega_0 > 2.52$. However, as A_2 becomes larger the solutions are numerically almost identical.

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